

IPS 2. — 30. 12. 2014.

1. Point M is given in the plane. a , b , c and d are four distinct lines passing through M , no two of which are perpendicular. A and B are two points on line a and b such that $MA = MB > 0$. C is a point on line c such that $\angle MAC = 90^\circ$, D is a point on line d such that $\angle MBD = 90^\circ$. Point W is the intersection of the perpendiculars from A to c and from B to d . Prove that MW and CD are perpendicular to each other.
2. Prove that if x , x^2 and x^n ($n \geq 3$ is a fixed integer) all have the same fractional part, then x is an integer number.
3. Finitely many line segments are given in the plane. The total length of the segments is less than $\sqrt{2}$. Prove that it is possible to find a unit square lattice such that all the given line segments are contained inside the unit squares of the lattice.

Time: 2,5 hours
Time for questions: 30 min
Each problem is worth 7 points.

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