1) Suppose that $G$ and $H$ are infinite graphs, and that $G$ is isomorphic to a subgraph of $H$ and $H$ is isomorphic to a subgraph of $G$. Must $G$ and $H$ be isomorphic?

2) Let $G$ be a 3-regular graph with $\chi'(G) = 3$ and suppose that there is a unique 3-edge colouring of $G$ (up to permuting the colours). Prove that $G$ has exactly 3 Hamilton cycles. Are there arbitrarily large graphs with this property?

3) A graph $G$ is $k$-list colourable if, whenever each vertex $v$ is assigned a list $L(v)$ of at least $k$ colours, it is possible to colour each vertex with a colour from its list so that adjacent vertices receive distinct colours. Construct a planar graph which is not 4-list colourable.

4) (a) Prove that every (not necessarily proper) 2-colouring of the edges of $K_{3n-1}$ contains $n$ vertex-disjoint edges of the same colour.

(b) Show that this does not hold for $K_{3n-2}$.

5) For which $n$ can you construct a planar graph $G$ with $|V(G)| = n$, $\delta(G) = 5$ and $\Delta(G) = 6$?

6) Prove that every intersecting family $\mathcal{F} \subset 2^{[n]}$ is contained in an intersecting family of $2^{[n]}$ of size $2^{n-1}$. 