Kombi problémamegoldó és kutatószeminárium

Various problems

2020. february 12.

1) Suppose that G and H are infinite graphs, and that G is isomorphic to a subgraph of H and H is isomorphic to a subgraph of G. Must G and H be isomorphic?

2) Let G be a 3-regular graph with $\chi'(G) = 3$ and suppose that there is a unique 3-edge colouring of G (up to permuting the colours). Prove that G has exactly 3 Hamilton cycles. Are there arbitrarily large graphs with this property?

3) A graph G is k-list colourable if, whenever each vertex v is assigned a list L(v) of at least k colours, it is possible to colour each vertex with a colour from its list so that adjacent vertices receive distinct colours. Construct a planar graph which is not 4-list colourable.

4) (a) Prove that every (not necessarily proper) 2-colouring of the edges of K_{3n-1} contains n vertex-disjoint edges of the same colour.
(b) Show that this does not hold for K_{3n-2}.

5) For which n can you construct a planar graph G with $|V(G)| = n, \delta(G) = 5$ and $\Delta(G) = 6$?

6) Prove that every intersecting family $\mathcal{F} \subset 2^{[n]}$ is contained in an intersecting family of $2^{[n]}$ of size 2^{n-1}.